

Feasible quantum optical experiment refuting noncontextuality for single photons

José L. Cereceda*

C/Alto del León 8, 4A, 28038 Madrid, Spain

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Abstract

Elaborating on a previous work by Simon *et al.* (Phys. Rev. Lett. **85**, 1783 (2000)) we propose a realizable quantum optical single-photon experiment using standard present day technology, capable of discriminating maximally between the predictions of quantum mechanics (QM) and noncontextual hidden variable theories (NCHV). Quantum mechanics predicts a gross violation (up to a factor of 2) of the noncontextual Bell-like inequality associated with the proposed experiment. This inequality can be used to experimentally demonstrate (modulo fair sampling) an all-or-nothing-type contradiction between QM and NCHV.

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*Electronic mail: jl.cereceda@teleline.es

An essential feature inherent in the world picture brought out to us by classical physics is that the measured values of any dynamical variable \mathcal{A} ascribed to an individual system are context independent, that is, they do not depend on what other commensurable variables $\mathcal{B}, \mathcal{C}, \dots$, are measured along with it. This central tenet is captured by the so-called non-contextual hidden variable (NCHV) theories which assume that the result of a measurement of \mathcal{A} depends solely on the choice of \mathcal{A} and on the state of the system being measured (the state being fully described by certain (noncontextual) hidden variables). While quantum mechanics (QM) satisfies this noncontextuality condition for the *expectation* value of the observables, the Bell-Kochen-Specker theorem [1-3] shows that noncontextuality is incompatible with the predictions that QM makes for *single* individual systems, provided the dimension of the associated Hilbert space is greater than two. Experimentally speaking, however, there have been relatively very few experiments testing the validity of the general assumption of noncontextuality, in contrast to the many experiments performed to test Bell's theorem [4] against *local* hidden variables (LHV) [5]. Indeed, to our knowledge, the recent two experiments performed by Michler *et al.* [6] are the first implementing explicitly a statistical test of NCHV versus QM. The second one of these experiments [6] is an “event ready” test of a Bell-like inequality involving only one particle, derived from the noncontextuality condition (see below for a brief discussion on this experiment). Also recently, in a very interesting paper Simon *et al.* [7], inspired by an argument by Cabello and García-Alcaine [8], developed a simple experimental scheme allowing a nonstatistical test of noncontextuality. This scheme works with single particles, and uses both their spin and translational degrees of freedom.

In this paper we present a practical quantum optical method for Simon *et al.*'s scheme which can be implemented with a suitable source of polarized single photons and a set of polarizing beam splitters and photodetectors. The original scheme in [7] exhibits a direct contradiction between NCHV and QM for *every* run of the experiment, so that, ideally, it would suffice a single particle to either prove or disprove noncontextuality. In practice, however, due to the imperfect detection efficiency and other nonidealities, it is not possible to attain the perfect correlations on which this all-or-nothing-type contradiction relies. Indeed, all we can do in a real experiment is to execute a sufficiently large number of runs in order to find out the experimentally significant ensemble averages. As we will see, an actual realization of the scheme in [7] requires determining the ensemble averages of the measurement outcomes of four appropriate *product* observables for single particles. It turns out that the assumption of noncontextuality implies an upper bound on the absolute value of a certain linear combination of these four ensemble averages (see Eq. (4) below), whereas the quantum-mechanical predictions can exceed this bound by a factor as large as 2 (which is the maximum possible theoretical violation of the inequality at issue). An empirical violation of this one-particle Bell-like inequality would then invalidate (modulo fair sampling) the noncontextuality as-

sumption. Furthermore, a violation of the inequality (4) by a factor approaching 2 would mean that we are refuting noncontextuality for *every* single particle of the ensemble (again, modulo fair sampling).

Before describing our experimental proposal for Simon *et al.*'s scheme, we briefly recall the logical contradiction between NCHV theories and QM entailed by the argument in [7] (see also [8]). Consider four two-valued observables Z_1 , X_1 , Z_2 , and X_2 ascribed to a single individual system, the measurement results for each observable denoted by ± 1 . It is assumed that, in a NCHV theory, the above observables have predefined noncontextual values either $+1$ or -1 , which are denoted as $v(Z_1)$, $v(X_1)$, $v(Z_2)$, and $v(X_2)$, respectively. Consider now an ensemble of systems for which the results of Z_1 and Z_2 are always found to be equal to each other, and the same for the results of X_1 and X_2 . Then it can be shown [7] that, for such an ensemble, NCHV predicts that $v(Z_1X_2) = v(X_1Z_2)$, where, by definition, for a NCHV theory $v(Z_1X_2)$ is equal to the product $v(Z_1)v(X_2)$. (Note that the considered ensemble can then equivalently be defined by the condition $v(Z_1Z_2) = v(X_1X_2) = 1$.) On the other hand, for the corresponding quantum-mechanical observables Z_1 , X_1 , Z_2 , and X_2 , we can find a two-qubit state [7] which is a joint eigenstate of the (commuting) product observables Z_1Z_2 and X_1X_2 , with eigenvalue $+1$ for both of them. Moreover, for this two-qubit state QM predicts that the measured value of the observable Z_1X_2 will always be opposite to the measured value of the (commuting) observable X_1Z_2 , in direct contradiction with the above NCHV prediction according to which $v(Z_1X_2) = v(X_1Z_2)$.

Let us now analyze the proposed interferometric arrangement depicted in Fig. 1 realizing the experimental scheme in [7]. A single photon linearly polarized along $+45^\circ$ enters the interferometer from the left in the spatial mode a . The polarizing beam splitter PS0 transmits only the horizontal (0°) polarization component of the incoming photon and reflects the vertical (90°) one, so that a photon polarized along the horizontal (vertical) direction will with certainty exit the upper (lower) output port of PS0, labeled as $u \rightarrow$ ($d \uparrow$) in the figure. PS1 and PS2 are polarizing beam splitters each transmitting the $+45^\circ$ polarization component while they reflect the -45° component, the diagonal polarization basis $\{|+45^\circ\rangle, |-45^\circ\rangle\} \equiv \{|\nearrow\rangle, |\searrow\rangle\}$ being related to the rectilinear polarization basis $\{|0^\circ\rangle, |90^\circ\rangle\} \equiv \{|\rightarrow\rangle, |\uparrow\rangle\}$ by

$$\begin{aligned} |\nearrow\rangle &= (1/\sqrt{2})(|\rightarrow\rangle + |\uparrow\rangle), \\ |\searrow\rangle &= (1/\sqrt{2})(|\rightarrow\rangle - |\uparrow\rangle). \end{aligned} \tag{1}$$

S1 and S2 are nonpolarizing 50–50 beam splitters with input modes u and d and output modes u' and d' , such that the translational state of a photon entering either S1 or S2 via one of its input modes will undergo the following unitary transformation

$$\begin{aligned} |u\rangle &\rightarrow (1/\sqrt{2})(|u'\rangle + |d'\rangle), \\ |d\rangle &\rightarrow (1/\sqrt{2})(|u'\rangle - |d'\rangle), \end{aligned} \tag{2}$$

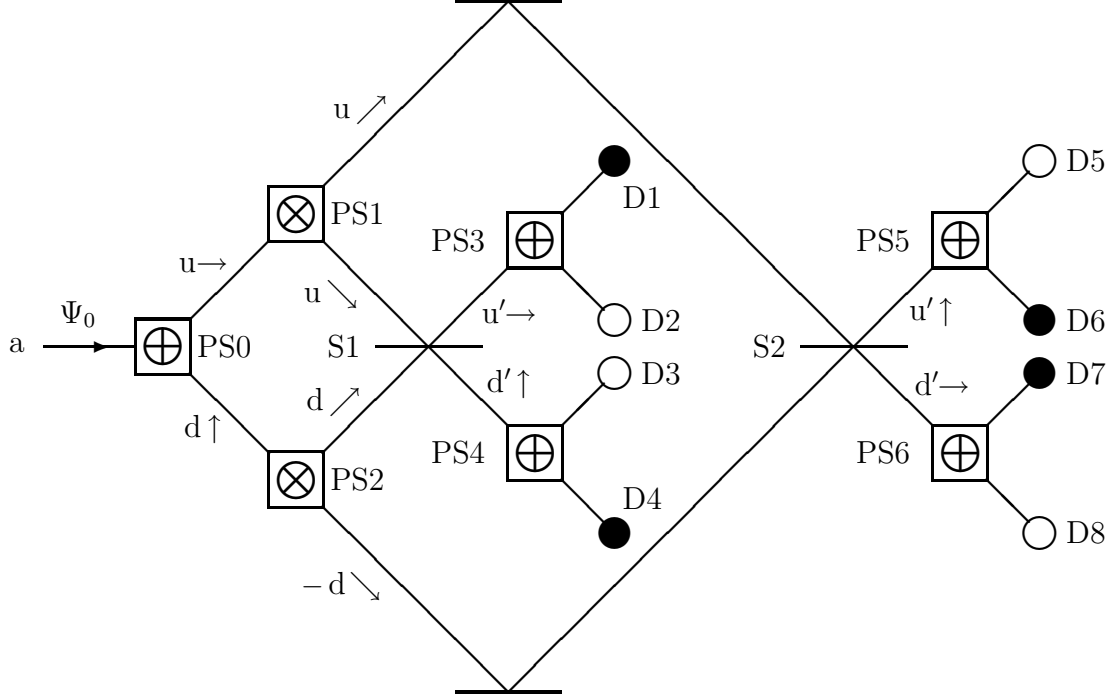


Fig. 1. Proposed experimental setup effecting the original scheme in [7]. Quantum mechanics predicts that the single photons (prepared in a suitable initial state $|\Psi_0\rangle$) can be detected only at the black-colored detectors, while NCHV predicts detections at the complementary set of white-colored ones (see the main text for details).

which is independent of the polarization state of the photon. Finally, PS3, PS4, PS5, and PS6 are polarizing beam splitters which measure the polarization of the incoming photon in the rectilinear basis $\{|\rightarrow\rangle, |\uparrow\rangle\}$, with the photon ending up in one of the photodetectors D1, D2, \dots , D8.

The observables to be considered in our interferometric setup are

$$\begin{aligned}
 Z_1 &= |u\rangle\langle u| - |d\rangle\langle d|, \\
 X_1 &= |u'\rangle\langle u'| - |d'\rangle\langle d'|, \\
 Z_2 &= |\rightarrow\rangle\langle\rightarrow| - |\uparrow\rangle\langle\uparrow|, \\
 X_2 &= |\nearrow\rangle\langle\nearrow| - |\searrow\rangle\langle\searrow|,
 \end{aligned} \tag{3}$$

where Z_1 and X_1 (Z_2 and X_2) involve the translational (polarization) degree of freedom of a single photon propagating inside the interferometer. It is easily verified that every single photon prepared in the initial state $|\Psi_0\rangle = |a\rangle|\nearrow\rangle$ will always yield +1 for the measured values of the product observables Z_1Z_2 and X_1X_2 . This follows from the fact

	D1	D2	D3	D4	D5	D6	D7	D8
Z_1X_2	-1	-1	-1	-1	+1	+1	+1	+1
X_1Z_2	+1	-1	-1	+1	+1	-1	-1	+1

Table 1: Results of the joint measurement of Z_1X_2 and X_1Z_2 , which is completed when the photon is detected at one of detectors D1, D2, \dots , D8.

that, upon interacting with PS0, the state $|\Psi_0\rangle$ is transformed into $\frac{1}{\sqrt{2}}(|u\rangle|\rightarrow\rangle + |d\rangle|\uparrow\rangle) = \frac{1}{\sqrt{2}}(|u'\rangle|\nearrow\rangle + |d'\rangle|\searrow\rangle)$. Moreover, a detection of the photon at any given detector corresponds to a *simultaneous* measurement of the product observables Z_1X_2 and X_1Z_2 . To see this, consider the case in that the photon is detected by D1, say. This immediately means that, for the detected photon, $X_1 = +1$ and $Z_2 = +1$, and then $X_1Z_2 = +1$ (see Fig. 1). On the other hand, the state of a photon just before reaching S1 consists of a superposition of the states $|u\rangle|\searrow\rangle$ and $|d\rangle|\nearrow\rangle$ so that a measurement of Z_1X_2 on this photon must necessarily give the result $Z_1X_2 = -1$. In Table 1 we display the values of Z_1X_2 and X_1Z_2 , as revealed by the detection of the photon at the various detectors D1, D2, \dots , D8. We note that, interestingly, the detection of a photon at any given detector provides the value of the product observable Z_1X_2 as a whole, without revealing any information about the values of Z_1 and X_2 separately [7].

Using the relations (1) and (2), it is straightforward to see that, according to QM, a photon entering the interferometer in the state $|\Psi_0\rangle$ can reach only detectors D1, D4, D6, and D7 (each with probability $\frac{1}{4}$). Note that such detectors entail opposite values of Z_1X_2 and X_1Z_2 . On the other hand, according to a NCHV theory, a photon fulfilling the property $Z_1Z_2 = X_1X_2 = +1$ must end up at one of detectors D2, D3, D5, or D8. So, for example, a photon existing in a NCHV state defined by $v(Z_1) = v(Z_2) = -1$ and $v(X_1) = v(X_2) = -1$ will end up at detector D8. So, theoretically, the interferometer setup of Fig. 1 exhibits conflicting predictions between QM and NCHV at a nonstatistical level, that is, for every single photon prepared in the initial state $|\Psi_0\rangle$.

Actually, however, any real experiment deals with a number of unavoidable imperfections and undesirable effects (such as noise, background, losses, low detector efficiencies, imperfect alignments, etc.) that make that only a relatively small fraction of the photons emitted by the source be detected. Moreover, it may happen that a photon is registered by a detector other than D1, D4, D6, or D7, due, for example, to faulty preparation procedure. So we need to average the measurement outcomes obtained in a large number of runs in order to get experimentally meaningful quantities (the validity of the fair sampling assumption taken for granted).

This having said, in what follows we concentrate on the part of the experimental setup of

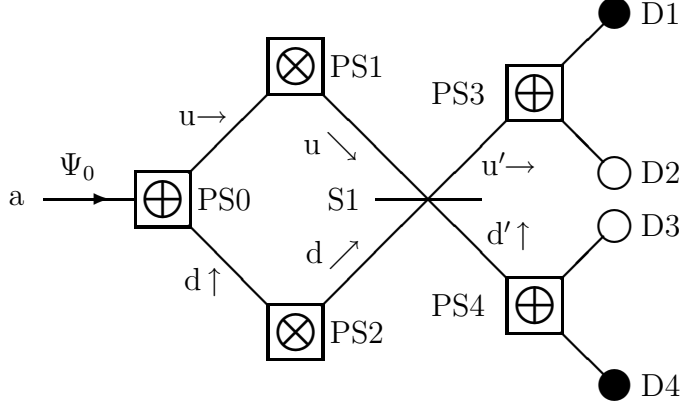


Fig. 2. Registering the counts at detectors D1, D2, D3, and D4, and averaging over a sufficiently large number of counts gives (via the value assignment of Table 1) the quantities $\langle Z_1 X_2 \rangle$ and $\langle X_1 Z_2 \rangle$. Thus, provided with the values of $\langle Z_1 Z_2 \rangle$ and $\langle X_1 X_2 \rangle$ which are obtained in a separate experiment, one can experimentally determine the left-hand side of the Bell-like inequality (4).

Fig. 1 that appears in Fig. 2, where we are interested only in detections at D1, D2, D3, and D4, while we ignore whatever detections at D5, D6, D7, and D8 (an entirely analogous argument develops if we instead restrict our attention to the latter set of detectors while ignoring the former one). For the setup appearing in Fig. 2, QM (NCHV) predicts detections in either D1 or D4 (D2 or D3) for an ensemble of photons prepared in the state $|\Psi_0\rangle$. Therefore, looking at detectors D1, D2, D3, and D4 already enables one to discriminate maximally between QM and NCHV. Consider the combination of product observables $C = Z_1 Z_2 + X_1 X_2 + X_1 Z_2 - Z_1 X_2$. In a NCHV theory C has the value $C_{\text{NCHV}} = v(Z_1)v(Z_2) + v(X_1)v(X_2) + v(X_1)v(Z_2) - v(Z_1)v(X_2)$, where, for example, $v(Z_1)$ in both the first and fourth terms assume the same value. Now, as each $v(Z_1)$, $v(Z_2)$, etc. is equal to either $+1$ or -1 , we have that $C_{\text{NCHV}} = \pm 2$. Thus, taking the ensemble average of $Z_1 Z_2$, $X_1 X_2$, $X_1 Z_2$, and $Z_1 X_2$, we arrive at the following inequality which must necessarily be fulfilled by any NCHV theory

$$|\langle Z_1 Z_2 \rangle + \langle X_1 X_2 \rangle + \langle X_1 Z_2 \rangle - \langle Z_1 X_2 \rangle| \leq 2. \quad (4)$$

This Bell-like inequality for single particles can be tested using the experimental arrangement of Fig. 2. Indeed, for the subensemble of photons detected at either D1, D2, D3, or D4, NCHV imposes an upper bound on $\langle C \rangle$ equal to $\langle C_{\text{NCHV}} \rangle^{\text{max}} = [\langle Z_1 Z_2 \rangle + \langle X_1 X_2 \rangle + \langle X_1 Z_2 \rangle - \langle Z_1 X_2 \rangle]_{\text{NCHV}}^{\text{max}} = +1 + 1 - 1 + 1 = 2$, in accordance with (4). Quantum mechanics, on the other hand, predicts an upper bound of $\langle C_{\text{QM}} \rangle^{\text{max}} = [\langle Z_1 Z_2 \rangle + \langle X_1 X_2 \rangle + \langle X_1 Z_2 \rangle - \langle Z_1 X_2 \rangle]_{\text{QM}}^{\text{max}} = +1 + 1 + 1 + 1 = 4$, which violates maximally the inequality (4) by a factor of 2.

Here we should mention that, as pointed out in [7], the observables Z_1 , X_1 , Z_2 , and X_2

appear in two *different* contexts, one involving the measurement of both Z_1Z_2 and X_1X_2 , and the other involving the measurement of both Z_1X_2 and X_1Z_2 . The setup of Fig. 2 performs a joint measurement of Z_1X_2 and X_1Z_2 . The observables Z_1Z_2 and X_1X_2 could be measured by means of an auxiliary device not shown in the figure. An empirical violation of the inequality (4) would disprove the class of NCHV theories which assume predefined noncontextual values of the observables Z_1 , X_1 , Z_2 , and X_2 . For the setup of Fig. 2, the maximal violation of the inequality (4) is attained when each member of the subensemble of detected photons is registered at either D1 or D4, which results in a direct contradiction between QM and NCHV for every photon of the original ensemble described by the state $|\Psi_0\rangle$, provided fair sampling holds good. It is worth noting, incidentally, that this all-or-nothing-type contradiction between QM and NCHV has been obtained for a quantum system pertaining to a four-dimensional (tensor product) Hilbert space. This is to be compared with the case of Bell's theorem, where it is necessary to consider at least *three* spin- $\frac{1}{2}$ particles (described in a (tensor product) Hilbert space of dimension eight) in order to get an all-or-nothing-type contradiction between QM and LHV [9,10].

The predicted quantum-mechanical violation of the noncontextual inequality (4) associated with the experimental scheme in Fig. 2, is greater than that obtained in previous schemes involving single particles described in terms of a four-dimensional Hilbert space [6,11] (for completeness, see also [12] where noncontextual Bell-type inequalities are derived for single particles of high spin). For example, the inequality tested in the second experiment in [6] is violated by QM by a factor up to $\sqrt{2}$. This inequality involves correlation functions for pairs of observables ascribed to a single particle (photon 1). However, for the experiment at issue, the determination of such correlation functions requires the previous detection of a second particle (photon 2) which is in turn correlated with photon 1, the registration of photon 2 at a given detector serving as a trigger event for the performance of the experiment on photon 1. Thus, the overall realization of the second experiment in [6] does involve two correlated photons in a necessary way, and so, being certainly an extremely valuable experiment, it cannot, in our view, be considered as a genuine single-photon one.

In conclusion, in this paper we have presented a feasible method for the original scheme of Simon *et al.* [7] which can be implemented with present day optical technology. Specifically, the interferometer setup sketched in Fig. 2 offers the basis for performing a real single-photon experiment capable of discriminating maximally (via the testable Bell-like inequality (4)) between the predictions of QM and NCHV.

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